$\alpha=180^\circ-A=110^\circ$
$\beta=180^\circ-B=48^\circ\ 50'$
$\gamma=180^\circ-C=85^\circ\ 10'$
$2\sigma=244^\circ$
$\sigma=122^\circ$
$\sigma-\alpha=12^\circ$
$\sigma-\beta=73^\circ\ 10'$
$\sigma-\gamma=36^\circ\ 50'$

To find $\log\tan\frac{1}{2}\delta$

$\log\sin(\sigma-\alpha)=9.3179$
$\log\sin(\sigma-\beta)=9.9810$
$\log\sin(\sigma-\gamma)=9.7778$
$\text{colog}\sin\sigma=0.0716$
$2|\underline{9.1483}$
$\log\tan\frac{1}{2}\delta=9.5742$

$\log\sin(\sigma-\alpha)=9.3179$
$\log\tan\frac{1}{2}\delta=9.5742$
$\log\tan\frac{1}{2}a=9.7437$
$\frac{1}{2}a=39^\circ$
$a=58^\circ$

$\log\sin(\sigma-\beta)=9.9810$
$\log\tan\frac{1}{2}\delta=9.5742$
$\log\tan\frac{1}{2}b=0.4068$
$\frac{1}{2}b=68^\circ\ 36'$
$b=137^\circ\ 12'$

$\log\sin(\sigma-\gamma)=9.7778$
$\log\tan\frac{1}{2}\delta=9.5742$
$\log\tan\frac{1}{2}c=0.2036$
$\frac{1}{2}c=57^\circ\ 58'$
$c=115^\circ\ 56'$

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

308. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Find the conditions that the roots of $x^2+px+q=0$ may not lie between -1 and $+1$.

I. Solution by J. A. CAPARO, University of Notre Dame, Notre Dame, Ind.

In the most general case let one of the roots be $+a$ and the other $-b$, then:

$$(x-a)(x+b)=0 \text{ or } x^2+x(b-a)-ab=0.$$

Comparing with $x^2+px+q=0$, $p=b-a$, $q=-ab$.

The conditions that the roots shall not lie between $+1$ and -1 are:

$$+a>1\ldots(1);\quad +b>1\ldots(2).$$

Multiplying, $ab>1$, but $ab=-q$, therefore $-q>1$ or $q<-1$. Also from (1), (2), $a-1>0$, $b+1>2$.

Multiplying, $ab-b+a-1>0$, or $-ab+(b-a)+1<0$, or $+q+p<-1$. The required conditions then are: $q<-1$ and $q+p<-1$.

II. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

Assuming that the equation has real roots, *i. e.*, that $p^2>4q$, find Sturm's functions. They are,

$$\left.\begin{array}{l} f(x)=x^2+px+q \\ f'(x)=2x+p \\ f_2(x)=p^2-4q \end{array}\right\}$$

For $+1$, we have, $f(x)=1+p+q$, $f_1(x)=2+p$, $f_2(x)=p^2-4q$.

For -1, we have, $f(x)=1-p+q$, $f_1(x)=-2+p$, $f_2(x)=p^2-4q$.

If there is no root between $+1$ and -1, there will be the same number of variations of signs in each of these. p^2-4q is always positive. We see that the number of variations will be the same if $f(1)$ and $f(-1)$, $f'(1)$ and $f'(-1)$ have the same sign, *i. e.*, $(1+q)^2-p^2>0$ and $p^2-4>0$, *i. e.*,

$$(1+q)^2>p^2>4.$$

They will also have the same number of variations if $f'(1)$ and $f'(-1)$ have opposite signs and $f(1)$ and $f(-1)$ are both negative, *i. e.*, $p^2<4$, $1+q\pm p<0$, or $p\pm q>1$.

Also solved by G. B. M. Zerr, V. M. Spunar, and G. W. Hartwell.

309. Proposed by PROFESSOR E. B. ESCOTT, Ann Arbor, Mich.

Solve, $bx^2+cy^2+az^2=ba^2+cb^2+ac^2$,
$cx^2+ay^2+bz^2=ab^2+bc^2+ca^2$,
$xyz=abc$.

Solution by the PROPOSER.

We see by inspection that $x=a$, $y=b$, $z=c$ is one set of solutions.

Then, put $x^2=a^2+u$, $y^2=b^2+v$, $z^2=c^2+w$. The first two equations become $bu+cv+aw=0$, $cu+av+bw=0$. Solving these, we have

$$\frac{u}{a^2-bc}=\frac{v}{b^2-ca}=\frac{w}{c^2-ab}.$$

Put these equal to s, and substitute

$$u=(a^2-bc)s,$$
$$v=(b^2-ca)s,$$
$$w=(c^2-ab)s,$$

in values of x^2, y^2, and z^2:

$$x^2=a^2+(a^2-bc)s,$$
$$y^2=b^2+(b^2-ca)s,$$
$$z^2=c^2+(c^2-ab)s.$$